

4.6 Use Congruent Triangles



Before You used corresponding parts to prove triangles congruent.

Now You will use congruent triangles to prove corresponding parts congruent.

Why? So you can find the distance across a half pipe, as in Ex. 30.

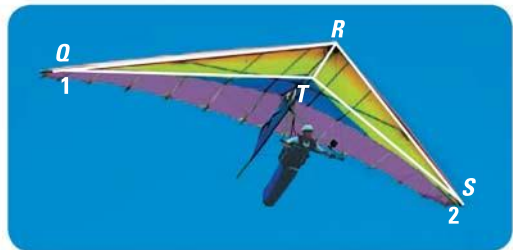
Key Vocabulary
 • **corresponding parts**, p. 225

By definition, congruent triangles have congruent corresponding parts. So, if you can prove that two triangles are congruent, you know that their corresponding parts must be congruent as well.

EXAMPLE 1 Use congruent triangles

Explain how you can use the given information to prove that the hanglider parts are congruent.

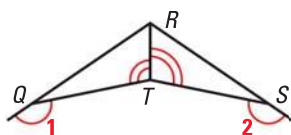
- GIVEN** ▶ $\angle 1 \cong \angle 2$, $\angle RTQ \cong \angle RTS$
PROVE ▶ $\overline{QT} \cong \overline{ST}$



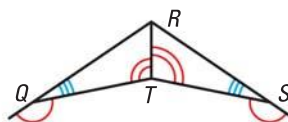
Solution

If you can show that $\triangle QRT \cong \triangle SRT$, you will know that $\overline{QT} \cong \overline{ST}$. First, copy the diagram and mark the given information. Then add the information that you can deduce. In this case, $\angle RQT$ and $\angle RST$ are supplementary to congruent angles, so $\angle RQT \cong \angle RST$. Also, $\overline{RT} \cong \overline{RT}$.

Mark given information.



Add deduced information.

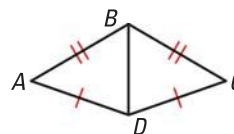


Two angle pairs and a non-included side are congruent, so by the AAS Congruence Theorem, $\triangle QRT \cong \triangle SRT$. Because corresponding parts of congruent triangles are congruent, $\overline{QT} \cong \overline{ST}$.

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✓ GUIDED PRACTICE for Example 1

1. Explain how you can prove that $\angle A \cong \angle C$.



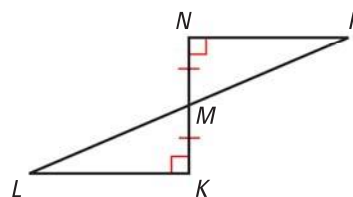
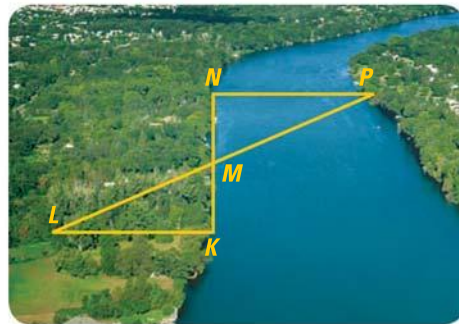
EXAMPLE 2 Use congruent triangles for measurement

INDIRECT MEASUREMENT

When you cannot easily measure a length directly, you can make conclusions about the length *indirectly*, usually by calculations based on known lengths.

SURVEYING Use the following method to find the distance across a river, from point N to point P .

- Place a stake at K on the near side so that $\overline{NK} \perp \overline{NP}$.
- Find M , the midpoint of \overline{NK} .
- Locate the point L so that $\overline{NK} \perp \overline{KL}$ and L , P , and M are collinear.
- Explain how this plan allows you to find the distance.



Solution

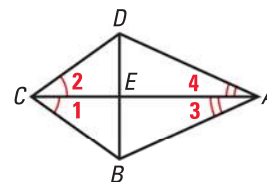
Because $\overline{NK} \perp \overline{NP}$ and $\overline{NK} \perp \overline{KL}$, $\angle N$ and $\angle K$ are congruent right angles. Because M is the midpoint of \overline{NK} , $\overline{NM} \cong \overline{KM}$. The vertical angles $\angle KML$ and $\angle NMP$ are congruent. So, $\triangle MLK \cong \triangle MPN$ by the ASA Congruence Postulate. Then, because corresponding parts of congruent triangles are congruent, $\overline{KL} \cong \overline{NP}$. So, you can find the distance NP across the river by measuring \overline{KL} .

EXAMPLE 3 Plan a proof involving pairs of triangles

Use the given information to write a plan for proof.

GIVEN $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

PROVE $\triangle BCE \cong \triangle DCE$



Solution

In $\triangle BCE$ and $\triangle DCE$, you know $\angle 1 \cong \angle 2$ and $\overline{CE} \cong \overline{CE}$. If you can show that $\overline{CB} \cong \overline{CD}$, you can use the SAS Congruence Postulate.

To prove that $\overline{CB} \cong \overline{CD}$, you can first prove that $\triangle CBA \cong \triangle CDA$. You are given $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. $\overline{CA} \cong \overline{CA}$ by the Reflexive Property. You can use the ASA Congruence Postulate to prove that $\triangle CBA \cong \triangle CDA$.

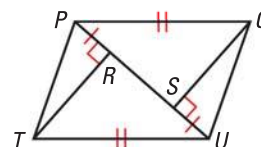
Plan for Proof Use the ASA Congruence Postulate to prove that $\triangle CBA \cong \triangle CDA$. Then state that $\overline{CB} \cong \overline{CD}$. Use the SAS Congruence Postulate to prove that $\triangle BCE \cong \triangle DCE$.

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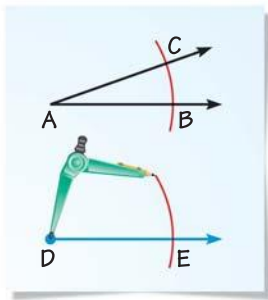
GUIDED PRACTICE for Examples 2 and 3

- In Example 2, does it matter how far from point N you place a stake at point K ? *Explain.*
- Using the information in the diagram at the right, write a plan to prove that $\triangle PTU \cong \triangle QUP$.



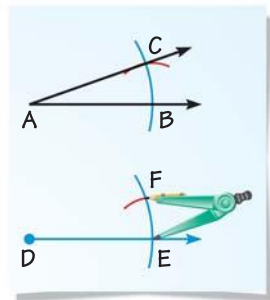
PROVING CONSTRUCTIONS On page 34, you learned how to use a compass and a straightedge to copy an angle. The construction is shown below. You can use congruent triangles to prove that this construction is valid.

STEP 1



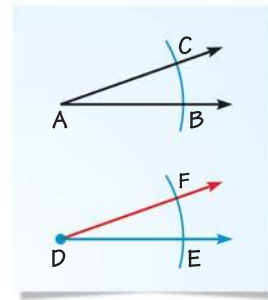
To copy $\angle A$, draw a segment with initial point D . Draw an arc with center A . Using the same radius, draw an arc with center D . Label points B , C , and E .

STEP 2



Draw an arc with radius BC and center E . Label the intersection F .

STEP 3



Draw \overrightarrow{DF} . In Example 4, you will prove that $\angle D \cong \angle A$.

EXAMPLE 4 Prove a construction

Write a proof to verify that the construction for copying an angle is valid.

Solution

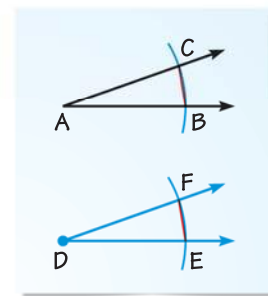
Add \overline{BC} and \overline{EF} to the diagram. In the construction, \overline{AB} , \overline{DE} , \overline{AC} , and \overline{DF} are all determined by the same compass setting, as are \overline{BC} and \overline{EF} . So, you can assume the following as given statements.

GIVEN $\triangleright \overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}$

PROVE $\triangleright \angle D \cong \angle A$

Plan for Proof

Show that $\triangle CAB \cong \triangle FDE$, so you can conclude that the corresponding parts $\angle A$ and $\angle D$ are congruent.



	STATEMENTS	REASONS
Plan in Action	1. $\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}$	1. Given
	2. $\triangle FDE \cong \triangle CAB$	2. SSS Congruence Postulate
	3. $\angle D \cong \angle A$	3. Corresp. parts of $\cong \triangle$ are \cong .



GUIDED PRACTICE for Example 4

- Look back at the construction of an angle bisector in Explore 4 on page 34. What segments can you assume are congruent?

4.6 EXERCISES

HOMEWORK KEY

○ = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 19, 23, and 31

★ = STANDARDIZED TEST PRACTICE Exs. 2, 14, 31, and 36

SKILL PRACTICE

EXAMPLES 1 and 2

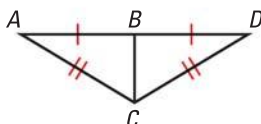
on p. 256–257 for Exs. 3–11

1. **VOCABULARY** Copy and complete: Corresponding parts of congruent triangles are ?.

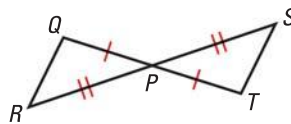
2. ★ **WRITING** Explain why you might choose to use congruent triangles to measure the distance across a river. Give another example where it may be easier to measure with congruent triangles rather than directly.

CONGRUENT TRIANGLES Tell which triangles you can show are congruent in order to prove the statement. What postulate or theorem would you use?

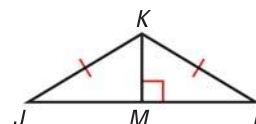
3. $\angle A \cong \angle D$



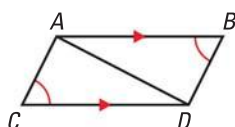
4. $\angle Q \cong \angle T$



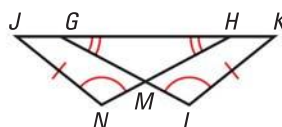
5. $\overline{JM} \cong \overline{LM}$



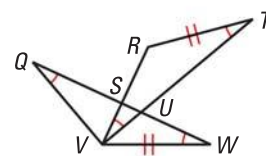
6. $\overline{AC} \cong \overline{BD}$



7. $\overline{GK} \cong \overline{HJ}$

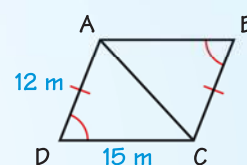


8. $\overline{QW} \cong \overline{TV}$



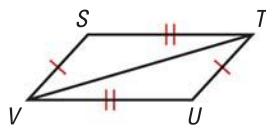
9. **ERROR ANALYSIS** Describe the error in the statement.

$\triangle ABC \cong \triangle CDA$ by SAS.
So, $AB = 15$ meters.

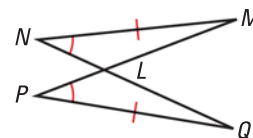


PLANNING FOR PROOF Use the diagram to write a plan for proof.

10. **PROVE** $\angle S \cong \angle U$



11. **PROVE** $\overline{LM} \cong \overline{LQ}$

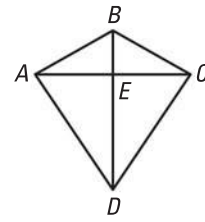


12. **PENTAGONS** Explain why segments connecting any pair of corresponding vertices of congruent pentagons are congruent. Make a sketch to support your answer.

13. **ALGEBRA** Given that $\triangle ABC \cong \triangle DEF$, $m\angle A = 70^\circ$, $m\angle B = 60^\circ$, $m\angle C = 50^\circ$, $m\angle D = (3x + 10)^\circ$, $m\angle E = \left(\frac{y}{3} + 20\right)^\circ$, and $m\angle F = (z^2 + 14)^\circ$, find the values of x , y , and z .

14. ★ **MULTIPLE CHOICE** Which set of given information does *not* allow you to conclude that $\overline{AD} \cong \overline{CD}$?

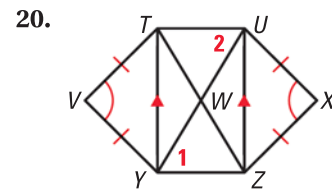
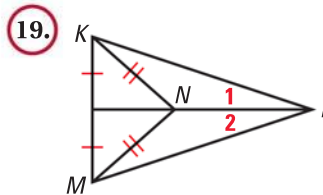
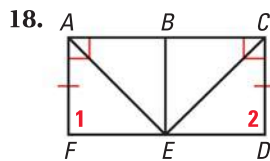
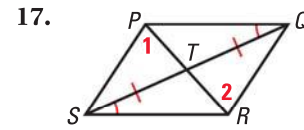
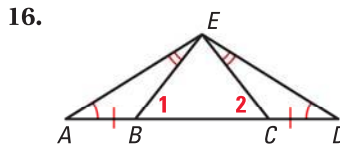
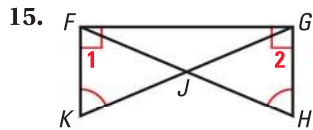
- (A) $\overline{AE} \cong \overline{CE}$, $m\angle BEA = 90^\circ$
 (B) $\overline{BA} \cong \overline{BC}$, $\angle BDC \cong \angle BDA$
 (C) $\overline{AB} \cong \overline{CB}$, $\angle ABE \cong \angle CBE$
 (D) $\overline{AE} \cong \overline{CE}$, $\overline{AB} \cong \overline{CB}$



EXAMPLE 3

on p. 257
for Exs. 15–20

PLANNING FOR PROOF Use the information given in the diagram to write a plan for proving that $\angle 1 \cong \angle 2$.

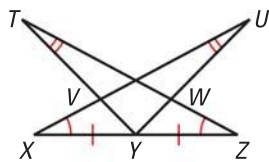


USING COORDINATES Use the vertices of $\triangle ABC$ and $\triangle DEF$ to show that $\angle A \cong \angle D$. Explain your reasoning.

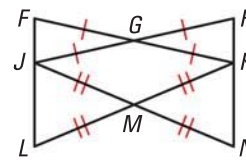
21. $A(3, 7)$, $B(6, 11)$, $C(11, 13)$, $D(2, -4)$, $E(5, -8)$, $F(10, -10)$
 22. $A(3, 8)$, $B(3, 2)$, $C(11, 2)$, $D(-1, 5)$, $E(5, 5)$, $F(5, 13)$

PROOF Use the information given in the diagram to write a proof.

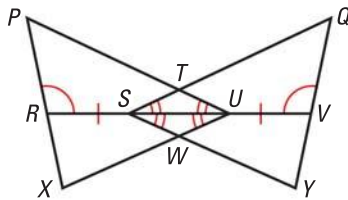
23. **PROVE** $\angle VYX \cong \angle WYZ$



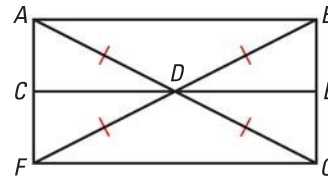
24. **PROVE** $\overline{FL} \cong \overline{HN}$



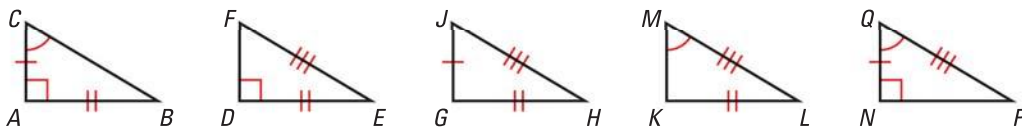
25. **PROVE** $\triangle PUX \cong \triangle QSY$



26. **PROVE** $\overline{AC} \cong \overline{GE}$



27. **CHALLENGE** Which of the triangles below are congruent?



PROBLEM SOLVING

EXAMPLE 2

on p. 257
for Ex. 28

28. **CANYON** Explain how you can find the distance across the canyon.

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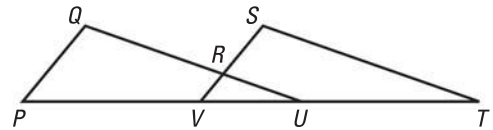


29. **PROOF** Use the given information and the diagram to write a two-column proof.

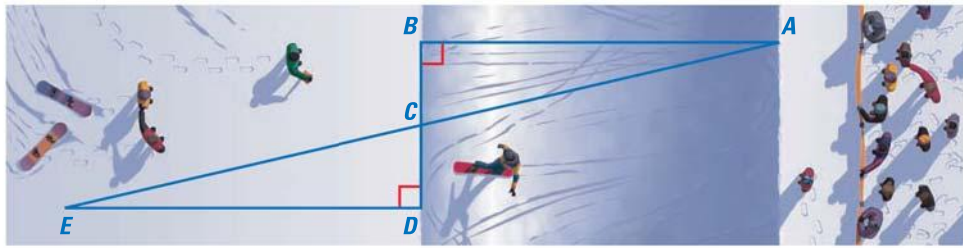
GIVEN ▶ $\overline{PQ} \parallel \overline{VS}$, $\overline{QU} \parallel \overline{ST}$, $\overline{PQ} \cong \overline{VS}$

PROVE ▶ $\angle Q \cong \angle S$

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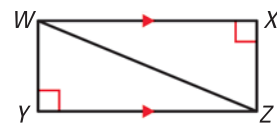


30. **SNOWBOARDING** In the diagram of the half pipe below, C is the midpoint of \overline{BD} . If $EC \approx 11.5$ m, and $CD \approx 2.5$ m, find the approximate distance across the half pipe. Explain your reasoning.



31. **★ MULTIPLE CHOICE** Using the information in the diagram, you can prove that $\overline{WY} \cong \overline{ZX}$. Which reason would *not* appear in the proof?

- (A) SAS Congruence Postulate
- (B) AAS Congruence Theorem
- (C) Alternate Interior Angles Theorem
- (D) Right Angle Congruence Theorem

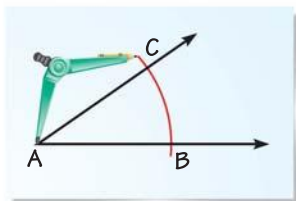


EXAMPLE 4

on p. 258
for Ex. 32

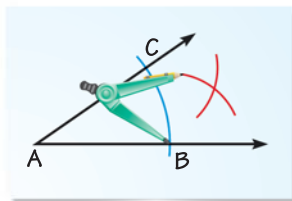
32. **PROVING A CONSTRUCTION** The diagrams below show the construction on page 34 used to bisect $\angle A$. By construction, you can assume that $\overline{AB} \cong \overline{AC}$ and $\overline{BG} \cong \overline{CG}$. Write a proof to verify that \overline{AG} bisects $\angle A$.

STEP 1



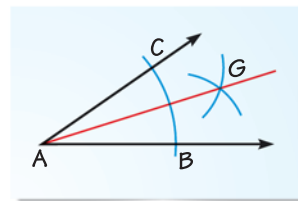
First draw an arc with center A . Label the points where the arc intersects the sides of the angle points B and C .

STEP 2



Draw an arc with center C . Using the same radius, draw an arc with center B . Label the intersection point G .

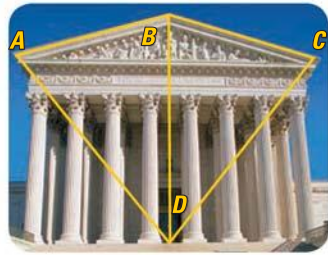
STEP 3



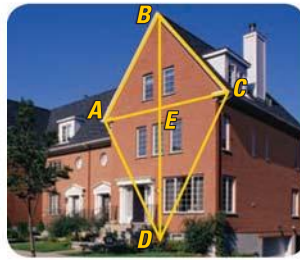
Draw \overline{AG} . It follows that $\angle BAG \cong \angle CAG$.

ARCHITECTURE Can you use the given information to determine that $AB \cong BC$? Justify your answer.

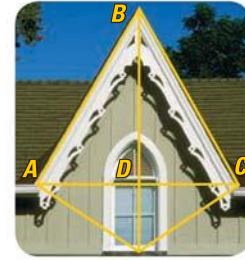
33. $\angle ABD \cong \angle CBD$,
 $AD = CD$



34. $\overline{AC} \perp \overline{BD}$,
 $\triangle ADE \cong \triangle CDE$

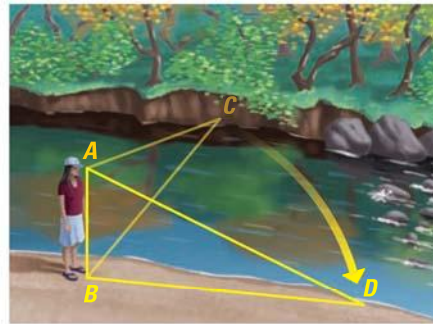
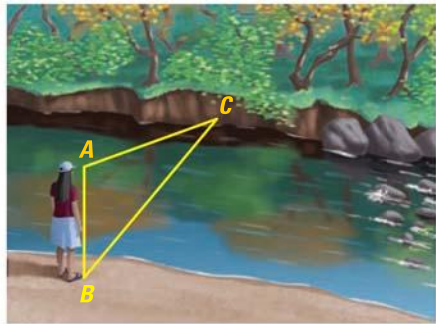


35. \overline{BD} bisects \overline{AC} ,
 $\overline{AD} \perp \overline{BD}$



36. **★ EXTENDED RESPONSE** You can use the method described below to find the distance across a river. You will need a cap with a visor.

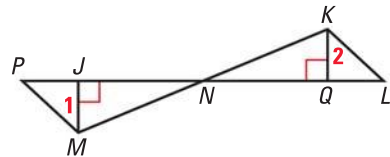
- Stand on one side of the river and look straight across to a point on the other side. Align the visor of your cap with that point.
- Without changing the inclination of your neck and head, turn sideways until the visor is in line with a point on your side of the stream.
- Measure the distance BD between your feet and that point.



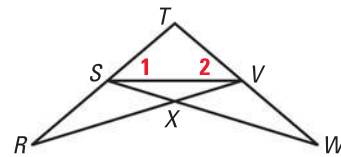
- What corresponding parts of the two triangles can you assume are congruent? What postulate or theorem can you use to show that the two triangles are congruent?
- Explain why BD is also the distance across the stream.

PROOF Use the given information and the diagram to prove that $\angle 1 \cong \angle 2$.

37. **GIVEN** $\overline{MN} \cong \overline{KN}$, $\angle PMN \cong \angle NKL$



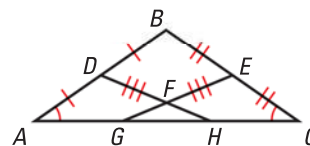
38. **GIVEN** $\overline{TS} \cong \overline{TV}$, $\overline{SR} \cong \overline{VW}$



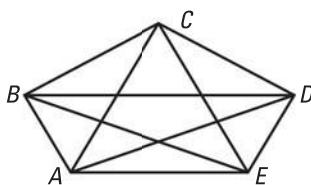
39. **PROOF** Write a proof.

GIVEN $\overline{BA} \cong \overline{BC}$, D and E are midpoints,
 $\angle A \cong \angle C$, $\overline{DF} \cong \overline{EF}$

PROVE $\overline{FG} \cong \overline{FH}$



40. **CHALLENGE** In the diagram of pentagon $ABCDE$, $\overline{AB} \parallel \overline{EC}$, $\overline{AC} \parallel \overline{ED}$, $\overline{AB} \cong \overline{ED}$, and $\overline{AC} \cong \overline{EC}$. Write a proof that shows $\overline{AD} \cong \overline{EB}$.



MIXED REVIEW

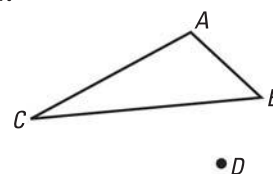
How many lines can be drawn that fit each description?

Copy the diagram and sketch all the lines. (p. 147)

41. Line(s) through B and parallel to \overleftrightarrow{AC}

42. Line(s) through A and perpendicular to \overleftrightarrow{BC}

43. Line(s) through D and C



PREVIEW

Prepare for
Lesson 4.7 in
Exs. 44–46.

The variable expressions represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles. (p. 217)

44. $m\angle A = x^\circ$
 $m\angle B = (4x)^\circ$
 $m\angle C = (5x)^\circ$

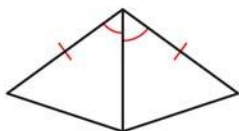
45. $m\angle A = x^\circ$
 $m\angle B = (5x)^\circ$
 $m\angle C = (x + 19)^\circ$

46. $m\angle A = (x - 22)^\circ$
 $m\angle B = (x + 16)^\circ$
 $m\angle C = (2x - 14)^\circ$

QUIZ for Lessons 4.4–4.6

Decide which method, SAS, ASA, AAS, or HL, can be used to prove that the triangles are congruent. (pp. 240, 249)

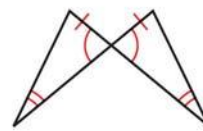
1.



2.



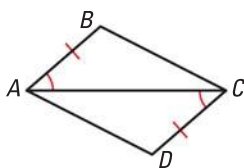
3.



Use the given information to write a proof.

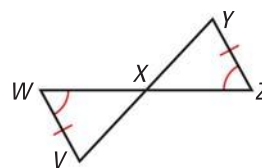
4. **GIVEN** $\angle BAC \cong \angle DCA$, $\overline{AB} \cong \overline{CD}$

PROVE $\triangle ABC \cong \triangle CDA$ (p. 240)



5. **GIVEN** $\angle W \cong \angle Z$, $\overline{VW} \cong \overline{YZ}$

PROVE $\triangle VWX \cong \triangle YZX$ (p. 249)



6. Write a plan for a proof. (p. 256)

GIVEN $\overline{PQ} \cong \overline{MN}$, $m\angle P = m\angle M = 90^\circ$

PROVE $\overline{QL} \cong \overline{NL}$

